ANALYSIS AND DESIGN TECHNIQUES FOR SHAPE MEMORY ALLOY MICROACTUATORS FOR SPACE APPLICATIONS

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Abstract

This paper presents the application of systematic model-based design techniques to the design of Shape Memory Alloy (SMA) actuators. Shape memory alloys are promising materials for (micro-)actuation in space applications, because of the relatively large deformations and forces that can be achieved. However, their complex constitutive behavior and the fact that several physical domains (electrical, thermal and mechanical) are involved makes the design effective SMA actuators with complex shapes and layouts challenging. Hence design optimization techniques are expected to play an important role in the further development of SMA actuators. This paper presents shape and topology optimization of SMA structures and shows the effectiveness of these design approaches by several representative examples.

Key words: shape memory alloys, R-phase transformation, topology optimization, element connectivity parameterization, shape optimization, bounded-but-unknown uncertainties.

1 INTRODUCTION

1.1 Shape Memory Alloys

In shape memory alloy (SMA) materials, a solid state phase transformation can take place under local temperature change and/or stress state variation. This transformation is associated with a transformation strain that can be utilized for actuation. SMA actuators are applicable in situations requiring relatively large deflections, combined with substantial mechanical loading [1, 2]. The temperature range required for actuation depends on the particular alloy. A drawback of macro-scale SMA actuators is that they have a comparatively slow response time in cyclic applications. This is caused by the fact that the actuator is controlled by temperature. Particularly cooling time is limiting the cyclic response, unless active cooling techniques are used (e.g. forced convection or Peltier elements). However, when scaling down to the micro-scale the cooling time decreases rapidly [3], because the surface/volume ratio and convection coefficient increase. Therefore, many future applications of SMA actuators in microsystems are expected, and several prototypes have already been demonstrated (e.g. [4, 5]).

SMA actuators are suitable for a variety of space applications, since their large power density offers compact and lightweight solutions, and their operation involves low accelerations and low voltage. Disadvantages are the mentioned limited bandwidth of SMA's, the need to consider the thermal environment and their power consumption. SMA actuators have successfully been applied on spacecraft in release and unfolding mechanisms for solar panels, such as those of the Hubble Space Telescope (1990) [6] as well as new microsatellites [7]. SMA-based systems are also being developed for the deployment of antennae or mirrors [8, 9]. Applications further include valves and apertures used in analysis instruments, such as the Ptolemy gas analyzer onboard of the Rosetta mission (2004) or the Material Adhesion Experiment used on the surface of Mars in the Pathfinder-Sojourner Mission (1997) [10]. SMA actuators have also been used in the Russian spacecraft Progress-40 (1989), the MIR space station, and many satellites [11].

As in many fields, a strong miniaturization trend can be observed in satellite technology, aimed at reducing device, launch and development costs [12, 13]. This has for example lead to the new classes of micro- and nanosatellites, with predictions of even smaller pico- and femto-satellites, enabled by developments in microsystem technology and active materials [14, 15]. Based on their intrinsic properties, SMA's have great potential for microactuation applications. Miniaturization of SMA's leads to improved bandwidth, and their actuation performance compares favorably with other active materials. Therefore it is expected that SMA micro-actuators can play an important role in future space applications.

1.2 SMA Design and Optimization

To realize the outlined potential of SMA's for space applications, improved design capabilities for SMA actuators are required. Many of the currently used SMA actuators rely on a relatively simple layout of the actuator, such as a wire-like or spring-shaped geometry. Clearly, understanding the behavior of a one-dimensional SMA wire or spring element is sufficient for effective design in these cases. This changes radically as soon as SMA actuators with more complex shapes and topologies are considered. As a consequence, it is necessary to understand and model the material behavior in a 2-D or even 3-D setting. The complex material behavior and the fact that several physical domains (electrical, thermal, mechanical) interact make it challenging to design effective SMA actuators with complex shapes and layouts. For that reason, the present paper focuses on automated design of SMA actuators using shape and topology optimization techniques.

In order to assist the design process of SMA structures, and to obtain a deeper understanding of the underlying phenomena that lead to the peculiar behavior of SMA materials, much research has already been done on the computational modeling of SMA constitutive behavior [16]. Surprisingly, however, there have been only a few publications reporting on combining computational SMA models with optimization techniques. Particularly the powerful combination of formal mathematical optimization with finite element analysis of the performance SMA structures has not been explored until recently [17].

1.3 Outline

Considering the state of SMA design optimization, there clearly is room for further generalization and extension of its applicability. This paper presents an overview of recent developments in the modeling and optimization of small-scale SMA actuators. The techniques outlined in this work are also applicable to further miniaturized SMA devices, which could play an important role in future microtechnology-enhanced space missions.

First, in Section 2, the constitutive model used in the design optimization is outlined briefly. Implications of the material model on the complexity of the optimization process are also discussed. Section 3 presents topology optimization of SMA actuators. This versatile technique can be used to find design concepts without specifying a particular design geometry in advance. After outlining problems encountered in the conventional density-based topology optimization formulation, the element connectivity parameterization approach is presented [18]. This approach circumvents difficulties of the conventional approach and enables topology optimization of SMA structures. Section 4 presents the shape optimization of SMA structures operated by Joule heating, applied to a miniature gripper showcase. A method to account for uncharacterized uncertainties in operating conditions and material properties is demonstrated. The fifth and final Section contains conclusions and outlines opportunities for further research.

2 SMA CONSTITUTIVE MODELING

2.1 Material

The shape memory effect considered is superelastic behavior due to the R-phase transformation in NiTi. Generally, SMAs exhibit hysteresis in the stress-strain-temperature behavior. However, in case of the R-phase transformation this hysteresis is particularly small, which is attractive for actuator applications. Experimental data obtained by Tobushi et al. [19] of the stress-strain-temperature behavior of a Ti-55.3wt%Ni alloy exhibiting the R-phase transformation is shown in Fig. 1.

It can be seen that the hysteresis between loading and unloading behavior is practically negligible in a certain temperature range, in this case between 328 and 343 K. Therefore, in the present model, this hysteresis is neglected.

2.2 Design Optimization Considerations

The SMA material model used in this work is particularly suited for design optimization. Design optimization generally is an iterative process that requires many evaluations of the analysis model. Thus, to make design optimization practically feasible, it is important to consider the efficiency of the analysis model. In the present work, as a first simplification, the dynamic response of the actuator is neglected, and a quasistatic simulation is performed. This is sufficient in many applications where the response speed is not critical.

A second aspect that is of great importance in design optimization - and particularly topology optimization - is the complexity of the sensitivity analysis. In history-dependent models, sensitivity analysis is much more involved than in the history-independent case [20]. The internal variables used in most of the existing SMA models render those models history-dependent. By neglecting the (rather small) hysteresis of the R-phase transformation, it turns out to be possible to eliminate internal state variables, resulting in a history-independent model. In that case, sensitivity analysis can be carried out at the end of the analysis, without the need to account for every increment during the evolution of the model over time.

2.3 Constitutive Model

A review of existing one-dimensional R-phase transformation models is given in [21]. A three-dimensional model has been developed to enable the analysis of general SMA structures. This SMA material model for superelastic behavior due to R-phase transformation in NiTi is based on three experimental observations:

1. The stress-strain-temperature curves (see Fig. 1).
2. The fact that the R-phase transformation is insensitive to hydrostatic pressure .
3. The fact that the R-phase transformation strain is isochoric.

Based on the first observation, a piecewise linear relation is defined that is fitted to the experimental results, yielding a one-dimensional model. Based on the second and third observations, a relevant scalar strain measure ε_e is defined related to the distortional strain energy. This effective strain ε_e is used to extend the one-dimensional model to the three-dimensional setting, in a way that is consistent with the experimentally observed R-phase transformation characteristics. The detailed derivation of this model and a description of the plane stress case can be found in [22]. The implementation has been verified against the available experimental data, as shown in Fig. 2. The model includes the observed main characteristics of the R-phase transformation and is presumed to capture the main effects of the constitutive behavior relevant for performing design optimization studies.

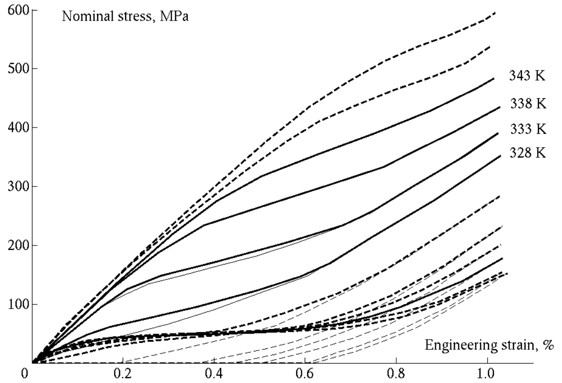

Fig. 1: Stress-strain curves at various temperatures from [19] of a Ni-Ti alloy. Thin lines are unloading curves.

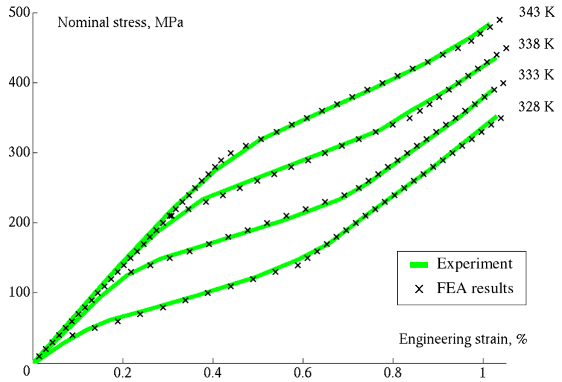

Fig. 2: Experimental stress-strain curves at various temperatures together with finite element results obtained using the proposed SMA material model.

3 SMA TOPOLOGY OPTIMIZATION

3.1 Difficulties in Conventional Topology Optimization

Topology optimization of SMA structures using the conventional topology optimization formulation suffers from several difficulties. In the conventional topology optimization approach, the material properties of every element in the finite element mesh are varied according to the value of the associated design variable. A widely used relation for linear elastic material is the SIMP model, where the Young's modulus E of each element is scaled using an exponential function of the element density [23]:

$$E = \rho^n E_0 \tag{1}$$

Here E_0 represents the nominal value of the Young's modulus, and the density design variable ρ varies between 0 and 1. The exponent n is used to introduce penalization of intermediate densities, i.e. to force the process toward well-defined black-white designs. The SIMP model preserves the convexity of the problem in case of compliance minimization, and also in other problems it has been used successfully [23]. However, for more complex nonlinear material models, the dependence between the material properties and the density design variable becomes ambiguous, and it is generally difficult to define a relation that leads to a converging optimization process. An incorrect choice easily results in a strongly nonconvex problem that quickly converges to an artificial local optimum. Moreover, for a given interpolation of the material model, the sensitivity analysis will require differentiation of that interpolation, which can become very involved for complex material models. Therefore, the conventional density-based approach is not attractive in cases with highly nonlinear constitutive models. For the present SMA model, these difficulties were also encountered, and the conventional density-based approach is not well suited.

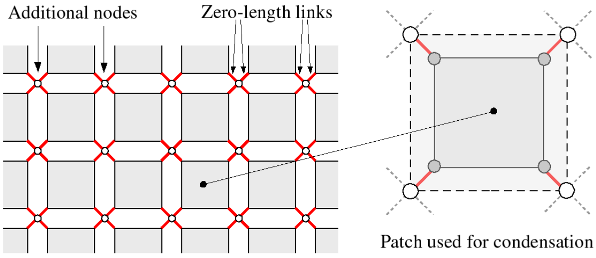

Fig. 3: Layout of elements and zero-length links (shown here with finite length) used in the element connectivity parameterization approach.

3.2 Element Connectivity Parameterization

In order to avoid the difficulties of the conventional density-based topology optimization formulation, the recently developed element connectivity parameterization (ECP) formulation [24, 25] is used here. Unlike the density based approach, the layout is not described by varying the densities and modifying the material properties of elements, but by varying the connectivity between elements. Computationally, this concept is implemented using zero-length links that connect the nodes of neighboring elements with additional nodes, see Fig. 3. The stiffnesses of the links connected to an individual element are controlled by a single design variable. Note that parameterization of element connectivity requires separation of elements, which increases the number of degrees of freedom of the problem significantly. This has been identified as a disadvantage of an earlier version of the ECP approach [18]. The present arrangement with additional nodes is chosen because it allows condensation of the element-link assemble (see Fig. 3), which reduces the number of degrees of freedom to that of the original mesh with directly connected elements. External forces and boundary conditions are applied to the additional nodes.

Because element connectivity instead of element properties are parameterized, clearly there is no need to define a material model interpolation. Therefore this approach is very suitable for topology optimization of SMA structures. Moreover, it turns out that the sensitivity analysis also is straightforward in case of ECP, because only the zero-length links are affected by the design variables. A more detailed discussion is outside the scope of the present paper, but can be found elsewhere [26].

3.3 Application to SMA Miniature Actuator

Using the SMA material model mentioned in Section 2, and the ECP topology optimization formulation briefly outlined in Section 3.2, topology optimization of an SMA actuator has been carried out successfully. The problem considered is the design of an SMA structure that can give the largest displacement difference for a given constant load, when the temperature of the structure is varied homogeneously between 328 and 343 K. Thus, the objective reads:

$$\max |U_{T=328\mathrm{K}} - U_{T=343\mathrm{K}}|. \tag{2}$$

The geometry of the design domain considered and the boundary conditions used are shown in Fig. 4, and because of symmetry only half the domain was used in the optimization process. The output displacement U considered in the

present case is the vertical displacement at the point where the force F is applied. A small non-design domain is used close to the point where the load is applied (shown in black in Fig. 4), to prevent that part from disappearing in the first step of the process. In contrast to usual topology optimization formulations, no mass constraint is used, as it turned out that the SMA actuator topology optimization problem has the property to select the required optimal amount of material by itself. A regular mesh consisting of 4000 standard four-noded isoparametric quadrilateral plane stress finite elements was used.
The optimizer used for the topology optimization is the method of moving asymptotes (MMA) [27]. Starting from a uniform design, and using an applied force of 100 N, after approximately 500 iterations the process converged and the design shown in Fig. 5 was obtained. The final stroke of this actuator is 0.55 mm.

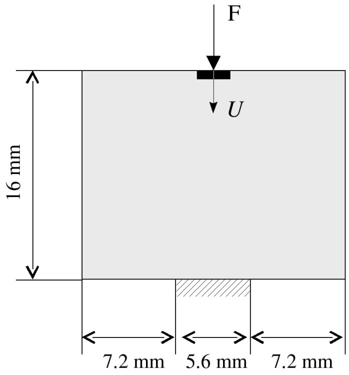

Fig. 4: Design domain and boundary conditions for the SMA topology optimization problem.

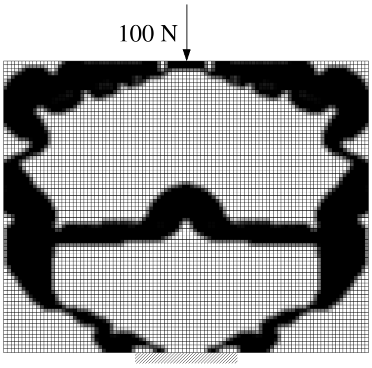

Fig. 5: Topology optimization result of an SMA thermal actuator, obtained using the ECP topology optimization formulation.

The developed topology optimization technique is able to generate designs for many different situations, e.g. by changing the shape of the design domain, the boundary conditions and the loading situation. It turns out that these factors can dramatically affect the optimal topology [28].

4 SMA SHAPE OPTIMIZATION

4.1 Problem Outline

Once the topology of a design has been chosen, its optimal geometry and dimensions can be determined using shape optimization. As a representative test case for shape optimization of SMA structures a microgripper is considered, as depicted in Fig. 6. This shape optimization problem will be briefly outlined here, a full description can be found in [17]. The SMA structure is prestressed by pinching the ends of the outer plates towards the middle. This prestress is necessary to achieve the SMA actuation effect under thermal cycling. The gripper is operated by applying a differential voltage over either the outer or inner plates, and a clamping force of 100 mN is applied in the closing configuration. The performance of this gripper is analyzed using two sequentially coupled electrical, thermal and mechanical FE analyses, one for the opening and one for the closing configuration. An adaptive incremental-iterative scheme is used to perform the nonlinear mechanical analysis. To find the optimal design, the geometry of the gripper is parameterized using 6 design variables, as shown in Fig. 7. In addition, the applied voltage is used as a design variable. The objective of the optimization problem is to maximize the range of motion of the gripper tips. The base temperature is taken to be 328 K, and the maximum temperature T of the gripper in operation is set to 338 K. In addition, the model used is only valid for effective strains ε_e up to 1%. Mathematically, this leads to the following optimization problem:

$$\begin{array}{lll} \text{max} & z^{\text{tip}}_{\text{open}} - z^{\text{tip}}_{\text{closed}} & \\ \text{s.t.} & T^{(i)} \leq 338 & i = 1 \ldots N,\ \text{open/closed} \\ & \varepsilon_e^{(i)} \leq 0.01 & i = 1 \ldots N,\ \text{open/closed} \end{array} \tag{3}$$

Here z^{tip} denotes the z-component of the tip position, and the constraints are evaluated separately for the opened and closed situations. N is the number of elements in the model. It can be seen that the maximum temperature and strain constraints lead to a large number of constraints. Therefore the Kreisselmeier-Steinhauser constraint aggregation technique [29] is used to reduce each set of constraints to a single constraint, leaving only 4 constraints.

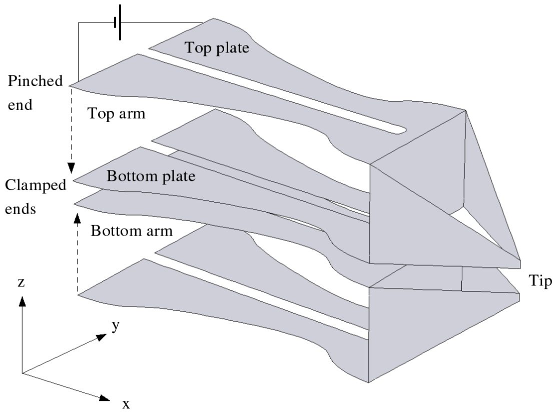

Fig. 6: Gripper geometry in the undeformed configuration.

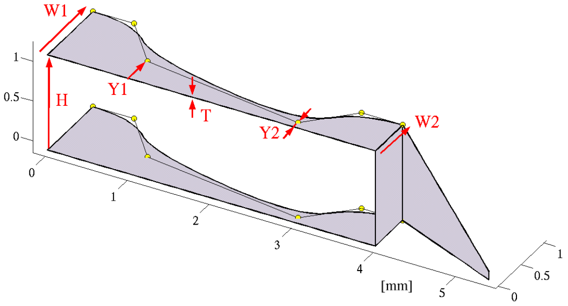

Fig. 7: Design parameterization of the gripper. Because of symmetry, only a quarter is considered.

4.2 Uncertainty-based SMA Shape Optimization

In many practical situations, not all factors that could potentially affect the performance of a device can be completely controlled, and their values are to some extent uncertain. In order to generate a reliable and robust actuator, it is important to consider these uncertainties in the design process. Various methods exist to deal with uncertainties in design optimization, and many of these methods make use of statistical information [30]. However, often such information is not available at the design stage, but simple bounds on the uncertain variables can usually be specified. This leads to the problem of design optimization involving so-called Bounded-But-Unknown (BBU) uncertainties [31]. In this approach, the objective f is optimized w.r.t. design variables $\mathbf{x}$, while anti-optimization is applied to the constraints $\mathbf{g}$ using the uncertainties $\boldsymbol{\alpha}$, in order to find the worst case settings of the uncertainty variables:

$$\begin{array}{ll} \min\limits_{\mathbf{x}} & f(\mathbf{x}) \\ \text{s.t.} & \mathbf{g}(\mathbf{x};\boldsymbol{\alpha}) \leq 0 \end{array} \tag{4}$$

where the uncertainties $\boldsymbol{\alpha}$ for each constraint are the maximizers of the anti-optimization problems given by:

$$\begin{array}{ll} \min\limits_{\boldsymbol{\alpha}} & g_i(\mathbf{x};\boldsymbol{\alpha}) \quad i = 1 \ldots n \\ \text{s.t.} & \mathrm{B}(\boldsymbol{\alpha}) \leq \mathbf{0} \end{array} \tag{5}$$

Here n is the number of constraints, and the operator B represents a set of bounds on the uncertainties. Because the anti-optimizations are nested within the main optimization, the process is very computationally intensive. Therefore an efficient cycle-based alternating anti-optimization strategy [32, 33] is used to solve this problem. In this approach, in combination with the multi-point approximation method [34], the number of anti-optimizations is drastically reduced in comparison to the rigorous approach, while convergence is still guaranteed.

After performing a sensitivity analysis at the deterministic optimum, the six most influential uncertainty parameters were chosen out of a set of ten. These are the thermal convection coefficient, the base temperature, and four parameters (E_A, E_R, E_0 and ν) related to the SMA material model [22]. After defining appropriate bounds on the uncertainties, BBU uncertainty-based design optimization has been carried out using a parallel computing framework, in order to reduce the time required for the process. In Fig. 8, the objective history during the optimization process of the deterministic and uncertainty-based shape optimization are compared. The optimized design accounting for uncertainties has a smaller tip displacement range than the deterministic optimum: 0.74 vs. 0.87 mm, as can be expected. Accounting for uncertainties leads to a more conservative design, which yields reduced performance but higher robustness and reliability. Also the optimal designs differ considerably, as shown in Fig. 9. The deterministic design is 12% higher, has a different shape, and the plates are 5% thinner compared to the design where the effect of uncertainties is considered. For more results and a further discussion of the uncertainty-based shape optimization of this SMA gripper the reader is referred to Ref. [35].

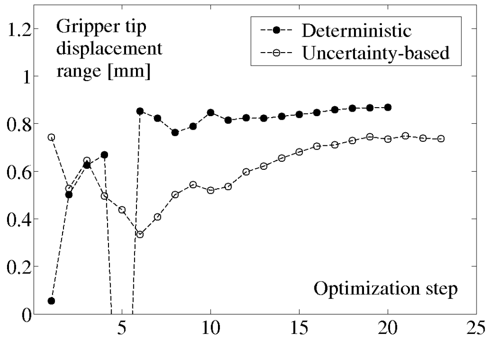

Fig. 8: Objective history during deterministic and uncertainty-based optimization.

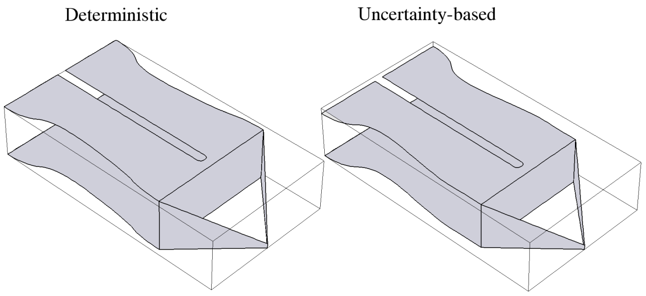

Fig. 9: Optimal geometries the SMA gripper (top arm shown) in case of deterministic and uncertainty-based shape optimization.

5 CONCLUSIONS

The present paper shows that topology and shape optimization techniques can be applied to design shape memory alloy actuators in an effective and systematic way. The material model used in this study is particularly suited for this application, because its history-independence enables efficient adjoint sensitivity analysis, which makes topology optimization feasible. In addition, the element connectivity parameterization approach avoids the need of defining a material interpolation and differentiation of the constitutive model, which is required in the conventional density-based topology optimization formulation. In combination with the MMA optimizer, an effective tool for the design of SMA actuator layouts has been realized, which is expected to be very helpful in particularly the conceptual design stage.

In addition, SMA shape optimization involving bounded-but-unknown (BBU) uncertainties has been performed. This approach does not require probabilistic information, which is often not available at the early design stage. Using the cycle-based alternating anti-optimization strategy in combination with parallel computing, the time required for the BBU optimization could be reduced to a practical level. It is expected that accounting for uncertainties will be very helpful in finding good SMA actuator designs for demanding applications. The outlined design techniques are applicable to a large variety of SMA microactuators, and they are expected to contribute in the design of actuators for future space applications.

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